# Multi-period Auctions for Network Resources

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***Abstract***

In recent years auctions have become increasingly popular for the efficient allocation of congested network resources, such as bandwidth. In this paper we consider a network provider performing auctions on end-to-end tunnels periodically, and users requesting bandwidth for long sessions (typically lasting for more than one auction period). Second-chance auctions have been recently proposed for preventing established sessions from being shut down due to an isolated short market perturbation. We propose and investigate three different types of second-chance auction mechanisms as well as optimal user strategies for them. Finally, simulation results demonstrate the advantages of second-chance auctions compared to standard multi-period PSP auctions.

## 1 Introduction and Related Work

Auctions are old and well-established mechanisms for achieving economic efficiency in resource allocation, as they assign commodities to those users who appreciate them most, and in this sense maximize "social welfare". Since the introduction of "smart markets" in the seminal paper [5], auctions have been repeatedly proposed also for the allocation of communication network resources. For a general survey on auction theory we refer e.g. to [11] or [1].

An auction scheme is described by a set of rules that specify the way of bidding, the determination of winning bidders and the price they are charged. An important game theoretic result, the "Revenue Equivalence Theorem", states that under rather general assumptions the outcome of all basic auction types is identical (see [11]). As a consequence, in our context usually some variant of the "Vickrey auction" [10] is used for the allocation of network resources, mainly due to its incentive compatibility (i.e. bidding truthfully is a dominant strategy). However, the classic Vickrey mechanisms can only be used for one single link in one single auction. Therefore the Vickrey auction has been generalized to the Generalized Vickrey Auction (GVA) in [6] for multiple units and to the Progressive Second Price auction (PSP) [4][3] for divisible resources. Incentive compatibility can be shown in both cases. As a further generalization, [9] presents two new schemes (Delta Auction, CHiPS) which deal with auctions in temporal and spatial dimension, while [8] implements the CHiPS principle as second-chance auction. Here, the basic idea is to give established users who have lost one auction out of a series a second chance in order to prevent their existing sessions from being shut down because of this isolated event.

Formulating this idea more generally, we consider a network which periodically performs auctions to allocate its resources, where the auction period is a natural consequence of the underlying technology, be it RSVP refreshment cycles or periodic reprovisioning of MPLS tunnels. In such a scenario, long sessions require the user to win a couple of subsequent auctions, whereas the loss of one single auction already causes the interruption of a running session. To overcome this "multi-period" problem we combine the CHiPS [9] and second-chance idea [8] and propose some new second-chance mechanisms for allocating bandwidth on end-to-end connections (e.g. MPLS tunnels).

The rest of this paper is structured as follows: Section 2 introduces the basic model of multi-period auctions, as well as a definition of four different multi-period auction mechanisms: the multi-period PSP auction, the simple second-chance auction, the second-chance auction with 2A1B and the second-chance auction with savings book, plus optimal strategies for these auction schemes. In section 3 we present simulation results for the various mechanisms and compare their performance. Finally, section 4 provides concluding remarks and further work.

# 2 Model and Mechanisms

In this paper we concentrate on allocating bandwidth on an end-to-end network path, e.g. an established MPLS (Multi Protocol Label Switching) tunnel or a reserved path under RSVP (Resource Reservation Protocol). These protocols provide a natural inherent periodicity in terms of refreshing reservations or provisioning, resp. Therefore, in such networks, auctions must be performed periodically if they are used as allocation mechanisms. We classify any such auction mechanism a "multi-period auction". The next subsections introduce some examples of multi-period auctions, starting with a serial concatenation of standard Progressive Second Price auctions.

## 2.1 Multi-period PSP auction

The Progressive Second Price (PSP) auction [4] is a generalization of the normal Vickrey auction rule for arbitrarily divisible resources. Whereas for Vickrey auctions, the winner is charged the bid of the highest bidding loser, the pricing rule here is slightly different. As in general bidders can demand different amounts of bandwidth units, the unit auction price now is a weighted average of the highest losing bids. It can be shown that the PSP auction is incentive compatible and converges into a Nash equilibrium. Although the PSP was originally designed as an open auction, the pricing rule (according to [3]) can also be used for sealed bid auctions, which seems to be more realistic in the case of network resources.

For our multi-period scenario, we propose the multi-period PSP auction as a reference mechanism as follows: Upon arrival, each user submits a two-dimensional bid $(d_i, \beta_i)$, where $d_i$ denotes his demand of bandwidth units and $\beta_i$ denotes his willingness to pay for one unit of bandwidth and one auction period[1]. Bids are collected until the next auction takes place, where eventually the resources are distributed between the competing users. This implicates the following problem: user sessions are in general longer than an auction period (i.e. the interval between the performance of two auctions). Consequently, a user has to win all auctions during his session. If he loses one of them, he cannot complete his session, which from the user's perspective clearly is a strong argument against using auction mechanisms for resource allocation at all. In order to prevent this, we require an auction rule that prefers users with already existing sessions.

## 2.2 The Simple Second-chance Auction

The first idea that prefers existing sessions was the CHiPS (Connection-Holder-is-Preferred Scheme) principle for the multi-link case [9]:

**CHiPS principle 2.1** *For an end-to-end connection comprising a series of single links, upon loss of an individual link auction, connection holders are not shut down immediately, but get a chance to increase their bid ex-posteriori.*

For the multi-period case, this idea is implemented in form of "second-chance auctions" defined as follows:

**Definition 2.2** *In the **simple second-chance auction** (SSC), each user with an existing session gets the chance to raise her bid after a lost auction, i.e. ex-post, to the market price, instead of closing down her session immediately.*

An interesting observation is that the auction price needs no longer be independent of the own bid as in any other second price auction (e.g. normal Vickrey auction, GVA, PSP): in order to continue, each user with an open session can raise her bid ex post if she has just lost the auction. But, at the same time she might be one of the highest losers, in case her bid was used for the calculation of the auction price (weighted average of the highest losers). Thus the price can depend on her own bid.

The optimal user strategy under SSC for the case of a user with an already existing session is formulated in the following Lemma:

**Lemma 2.3** *Under SSC, it is a dominant strategy for each user having an established session to bid* 0 *in each following auction.*

**Proof:** Suppose bidder $i$ has won the auction $\tau$ and wants to continue his session, i.e. she also wants to win auction $\tau + 1$. Let $\beta_{i,\tau}$ denote user $i$'s price component of the bid and $p_{i,\tau}$ the result (unit price) of auction $\tau$. Due to the design of the second-chance auction rules, a user having an already running session can be sure to win also the following auction (at least by taking the second-chance option). From [4], Lemma 3, it follows that the auction price increases monotonically (though the monotonicity is not necessarily strict) with the submitted price component $\beta_{i,\tau+1}$, i.e.

$$p_{i,\tau+1}(0) \leq p_{i,\tau+1}(\beta_{i,\tau+1}) \qquad \forall\ \beta_{i,\tau+1} > 0 \qquad (1)$$

Thus, a user with a running session cannot do better than submitting 0 as bid, as this guarantees the minimal price for her. □

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[1] The users are assumed to know the time interval between two auctions.

Note that this lemma suggests that under SSC, auction results can become rather low. We will provide some more detailed simulative evidence for that in section 3.

## 2.3 Second-chance Auction with 2A1B

Now we introduce an additional auction rule, in order to reduce the signalling overflow (raise the bid ex post) and to try to solve the problem of the too low prices. This is formulated in the following 2A1B-principle (Two Auctions–One Bid):

**2A1B principle 2.4** *Each bid is at the same time valid ex-post for the last auction and regularly for the imminent one.*

This principle forces a user immediately to bid at least the current auction price, as otherwise he would lose the last auction ex-post. Combining this rule with SSC yields a new auction scheme called "second-chance auction with 2A1B" (SC-2A1B). Similarly to lemma 2.3, an optimal bidding strategy for users with an existing sessions can be derived.

**Lemma 2.5** *Suppose the bandwidth over one link is allocated through the SC-2A1B. Then it is a dominant strategy for each user having an open session to bid the market price in each following auction.*

**Proof:** The proof is a straightforward generalization of Lemma 2.3 and therefore omitted. □

Hence, each user with an existing session will still submit the lowest possible bid instead of revealing some information about her real preference situation. As this is still not satisfying, an additional auction rule is presented in the next subsection.

## 2.4 The Second-chance Auction with Savings Book

Additionally to the rules of SC-2A1B, a new control variable $\epsilon_{i,\tau}$, called the "savings book", is introduced. The "second-chance auction with savings book" (SC-SB) works as follows: If a user wins an auction definitely, i.e. without resorting to the second-chance option, the savings book $\epsilon_{i,\tau}$ is increased by the difference between market price and user bid. If a user with a running session loses an auction, the rebidding (i.e. the bid for the next auction) is limited by the savings book, i.e. the bid can only be raised ex-post if the difference between market price and original (losing) bid is smaller than her account balance. In this case, the savings book is subsequently diminished by the bid increase. Otherwise the user loses the auction. The complete SC-SB mechanism can be summarized as follows:

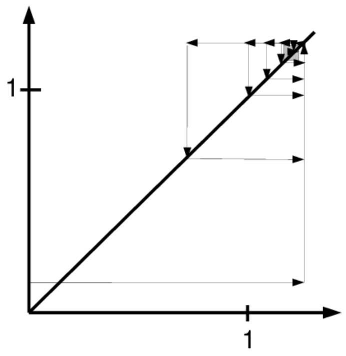

Figure 1: Simulative Nash equilibrium for SC-SB

- $\epsilon_{i,t_0} := 0$ for $\forall$ $i$.
- $\beta_{i,\tau} \geq p_{i,\tau}$ (user $i$ wins the auction definitely): $\epsilon_{i,\tau+1} := \epsilon_{i,\tau} + (\beta_{i,\tau} - p_{i,\tau})$.
- $\beta_{i,\tau} < p_{i,\tau}$ (user $i$ with running session loses): $\beta_{i,\tau+1} \leq \beta_{i,\tau} + \epsilon_{i,\tau}$ and $\epsilon_{i,\tau+1} := \epsilon_{i,\tau} - (\beta_{i,\tau+1} - \beta_{i,\tau})$.

As a consequence, a user with an already existing session can no longer be sure to win the auction. If she always submits the lowest possible bids, she risks eventually not to be allowed to use the second chance. Thus the savings book rule provides an incentive to submit higher bids than under SC-2A1B.

Deriving optimal user strategies is very difficult in this setting. At least we have found a Nash equilibrium via simulation of a static scenario (250 single auctions, constant user arrival rate, identical distributed user demand, identical distributed user session holding time, identical distributed user valuation) implemented in AMPL/CPLEX [2]: Considering every 100th user to be a "black user" who behaves differently from all other users, we assume that each user $i$ bids $\beta_{i,t} = \alpha_i * v_i$ for auction $t$ ($v_i$ denoted his one period valuation). All normal users $j_1, j_2, ...$ use identical $\alpha_{j_1} = \alpha_{j_2} = ... = 1$, whereas the $\alpha$ of the black user is varied in order to find out her optimal strategy with respect to the user's revenue. Having found the optimal $\alpha$, we start a new simulation cycle where all normal users use exactly this $\alpha$, whereas the black user again may behave differently etc. Figure 1 illustrates how, after a couple of simulation cycles, a Nash equilibrium with $\alpha \approx 1.3925$ is reached. As this value is different from 1, also SC-SB is not yet incentive compatible, but at least the problem of too low prices is solved, as we will see in the next section.

Figure 2: Demand in each auction

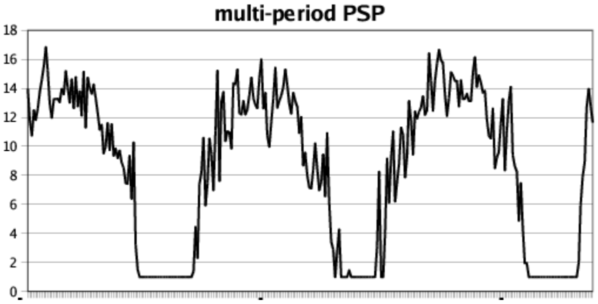

Figure 3: Multi-period PSP auction

# 3 Simulation Results

In this section we present simulation results for all four presented multi-period auction flavours. Following [8], we have chosen the linear-additive valuation metric where the valuation of a partially completed session increases linearly with the session holding time[2]. Thus an interrupted session still has some value to the user. Our simulation model and parameters can be summarized as follows:

- Simulation of 3 days with an auction interval of 15 minutes (288 single auctions).
- System capacity (bandwidth) $C = 100$.
- User arrival rate depending on time-of-day between 6 and 34.
- Uniformly distributed user demand $d_i \in [2, 4]$.
- Uniformly distributed user session holding time of $[7.5, 75]$ minutes.
- Uniformly distributed user valuation of $v_i \in [1, 20]$.

All programs have been written and implemented in the AMPL/CPLEX [2] language.

Figure 2 illustrates the demand distribution for each single auction, displaying an obvious time-of-day effect, where in peak times the demand is nearly 4 times as high as the system can serve, whereas in low demand periods only half of the bandwidth can be allocated at all.

For the multi-period PSP auction, the resulting prices are drawn in figure 3. The price is highly correlated with the underlying demand (Corr = 0.998). In peak times (total demand nearly 400 units of bandwidth) the prices are close to 17, whereas in times with smaller demand the price equals 1, i.e. the reserve price. The average price over the 288 auctions equals 8.53.

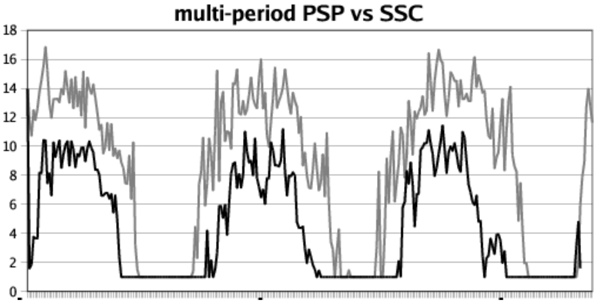

Figure 4: Multi-period PSP (grey) and SSC (black)

Figure 4 shows the prices of a SSC (black) in comparison with the multi-period PSP auction (grey). According to lemma 2.3 each established user bids 0. The price of the SSC is correlated (Corr = 0.79) with that of the normal multi-period PSP, but always considerably smaller than the latter one. The average price of the SSC (4.43) is much smaller than that of the multi-period PSP (8.53). Therefore the SSC seems to have a bad effect on the price, leading to a strict user gain (lower prices plus guarantee for session completion) at the cost of the auctioneer. The positive aspect is that no session has been interrupted, as we will see later.

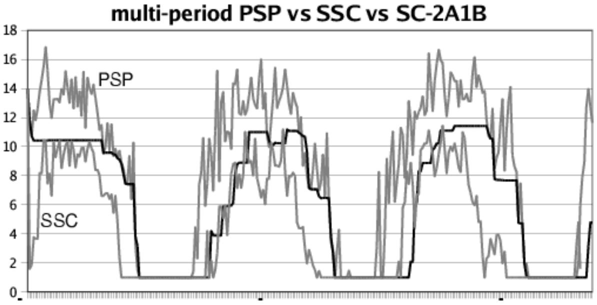

Figure 5: Multi-period PSP (grey), SSC (grey) and SC-2A1B (black)

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[2] Note that [8] contrasts this particular metric with an exponential-multiplicative and an all-or-nothing (inelastic) metric.

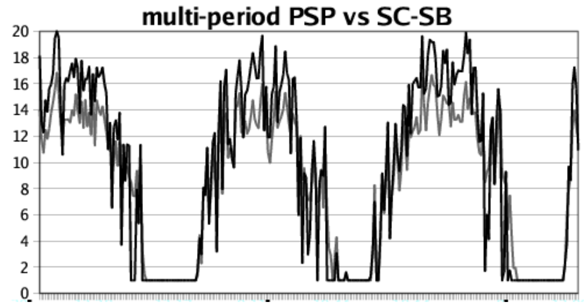

Figure 6: Multi-period PSP (grey) and SC-SB (black)

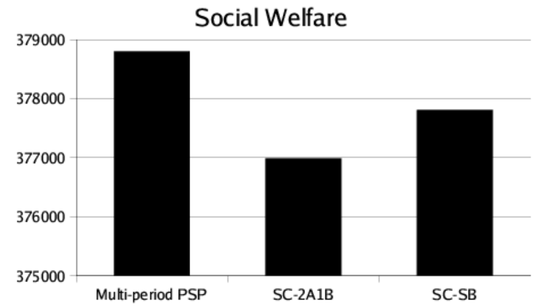

Figure 7: Comparison of social welfare of the different auctions

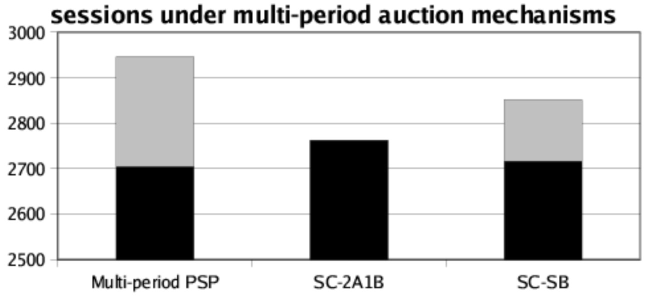

Figure 8: Comparison of completed (black) and interrupted (grey) sessions of the different auctions

Including the 2A1B-principle 2.4, Figure 5 shows as simulation result that the price of the SC-2A1B is higher than with SSC but still below PSP. The average auction price increases to 6.05. As the optimal strategy of users with an existing session is to bid the market price, the auction price becomes a step function, with the correlation between SC-2A1B and multi-period PSP decreasing to 0.68. Note that the price of SC-2A1B follows the PSP price with a delay of some periods in times of increasing demand but decreases almost immediately as soon as demand decreases. The former delay results from the lower bids of the user with an existing session compared to the bids in the multi-period PSP case. The price only rises when new users with total demand exceeding the total capacity have a higher valuation than the market price. On the other hand the price decreases nearly immediately as the users with existing sessions are likely to be winners without using their second-chance option as soon as the demand is decreasing. This enables them to lower their next bid immediately according to the current market price.

We have also simulated the SC-SB. By using $\alpha = 1.3925$ as dominant user strategy we obtain auction prices as shown in figure 6. The grey line shows the auction prices of the normal multi-period PSP and the black one the auction prices of the SC-SB which are no longer strictly smaller than for PSP and additionally show a larger fluctuation. The correlation between the multi-period PSP and the SC-SB however has a high value of 0.96, while the average of the auction prices increases to 9.34.

Finally we compare three of the multi-period auctions (normal multi-period PSP auction, SC-2A1B, SC-SB) concerning social welfare and proportion of completed sessions. Figure 7 shows the social welfare, defined as the sum over all satisfied user valuations aggregated over all auction periods. Here, the multi-period PSP result outperforms both the SC-2A1B and the SC-SB, but only by roughly 0.5%. Therefore the multi-period PSP auction is considered to be the best alternative from the point of a social planner, while the other two variants end up not far behind.

The number of completed (black) vs interrupted sessions (grey) is shown in figure 8. Here, the multi-period PSP auction has a success ratio (proportion of completed sessions) of only 92%, in contrast to 100% for the SC-2A1B (completes all sessions) and 95% for the SC-SB.

Summarizing our simulation results, we conclude that second-chance auctions are to be preferred over multi-period PSP, mainly due to the higher proportion of completed sessions. Among the presented variants, the choice between SC-2A1B and SC-SB mainly reflects the trade-off between realistic market prices and guaranteed session completion despite of varying market conditions.

# 4 Conclusions and Future Work

The main problem addressed in this paper concerns the design of a periodic auction mechanism for user sessions with long holding times which allow the users to successfully complete started sessions even in the case of major market turbulences. In this context, second-chance auctions have turned out to be a valuable multi-period allocation mechanisms for networks resources. We have presented three flavors of second-chance auctions and investigated their dominant strategies. Finally, simulation results illustrate the resulting auction prices as well as the social welfare and the proportion of completed sessions.

Current and future work extends the game-theoretic analysis of this new auction type, especially with respect to a more general approach to the question of dominant strategies and Nash equilibria. The most important focus, however, is the development of a second-chance auction, which is provably incentive compatible.

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