An Upper Bound to the Loss Probability in the Multiplexing of Jittered Flows

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Abstract. In the context of Diffserv networks some services should be characterized by end-to-end quantitative QoS guarantees. In order to provide such guarantees to single flows (or flow aggregates), the end-to-end analysis of delay and loss performance in a Diffserv domain is needed. The impact of jitter should be considered in the performance analysis at the successive nodes along the path of a flow (flow aggregate). Worst-case analysis is a solution to provide deterministic quantitative guarantees, at the price of very low efficiency. As an alternative this paper proposes a probabilistic approach, aimed at providing statistical quantitative guarantees and achieve higher efficiency. The proposed analytical approach is based on the insertion of a discarding device before the FIFO queue, called "dropper". The purpose of the dropper is to avoid the analysis of the congestion at the burst level in the queue, allowing for the application of an analytical result derived for packet scale conflicts in the *modulated ND/D/1* queue. Simulations are presented that validate the analytical bound. Finally numerical results are provided to evaluate the efficiency of the bound in an admission control scheme.

1 Introduction

Providing QoS guarantees in a packet switched network means also to be able to keep the end-to-end loss level below a target value. In order to do that, a method is essential to evaluate the loss probability at any network node, or at least to provide an upper bound for it, given a certain characterization of the traffic mix feeding the network. On the basis of such a method, one can build an Admission Control mechanism, centralized or distributed, able to ensure that the end-to-end loss probability along any generic path does not exceed the admissible value.

One major problem when dealing with performance analysis in multistage networks is to take into account the effect of jitter. As the traffic flow originated by the generic i–th source crosses several nodes (i.e. multiplexer) through the network, let f_i^k represent the arrival process relevant to this flow at the k-th node along his path. If the sources are directly connected to the network f_i^1 characterizes the original i-th source

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flow. In general the arrival process f_i^k (k>1) has different characteristics from f_i^1, because of the jitter introduced by the previous k-1 multiplexing processes. In the literature many methods are proposed to analyze the multiplexing performances in different cases depending on the characterization of the flows sources, i.e. f_i^1 in our notation. Such methods can only be applied to the nodes located at the edge of the network, i.e. before any jitter is introduced onto the flows. The problem of performance analysis in the internal nodes, or equivalently the problem of accounting for the effect of jitter, is faced in the literature in the following ways:

– the problem is neglected basing on the conjecture of "negligible jitter"[1];
– the effect of jitter is taken into account in a deterministic worst-case fashion [4] [5] [6].

The first approach can be applied only in a restricted class of cases, i.e. when the involved delays are "small" (compared to a poissonian scenario of equivalent intensity). The second approach, though extensively addressed in recent papers, in general leads to very low efficiency. In particular it has been shown ([4][5][6]) that the deterministic worst-case "explodes" when the number of multiplexing stages increases, accordingly the achievable efficiency diminishes dramatically.

In this paper an alternative approach is proposed, where the effect of jitter, assumed as non-negligible, is taken into account looking for statistical guarantees instead of deterministic worst case guarantees. The approach is mainly based on an analytical result obtained in [1] and [2] for the *modulated N·D/D/1 queue* and successive manipulation given in [3]. The basic idea is to reduce to the analysis of the per-flow arrival patterns in a sliding time window of opportune length: in the internal of such a window the effect of conflicts on the queue state can be evaluated by means of the above mentioned analytical result. The effect of jitter is taken into account in terms of the maximum packet clumping, which has an impact on the maximum number of arrivals in the considered time window. The effect of conflicts at a time scale larger than the considered window is accounted for by means of a discarding device, called "dropper", which is inserted before the queue. Its function is to filter such large-time-scale conflicts thus isolating the conflicts inside the considered time window. Because of the presence of the dropper the overall multiplexer system is non-work-conserving, therefore it can be assumed that the upper bound to loss probability derived for such a system works for the simple FIFO queue multiplexer too. In this sense the dropper is to be considered simply as a "virtual" device which allows for the analytical derivation of the bound.

The rest of the paper is organized as follows: In section 2 the complete analytical approach is presented. In section 3 the results are compared with simulations. Section 4 discusses the application to a sample case study. Finally conclusions are discussed in section 0 along with some directions for further study.

2 The Analytical Approach

2.1 The Model

Consider the multiplexer model of Fig. 1. It represents a generic network node (multiplexer) Ω in the network. Before reaching node Ω, the traffic flow originated by the generic *i*-th source, denoted by $f_{i,s}$, crosses a certain number of previous multiplexing stages which introduce a variable queuing delay (jitter), whose cumulated maximum value will be denoted by Δ_i. The number of different flows entering the node is *I*. The arrival process at node Ω relevant to *i*-th source will be denoted by f_i.

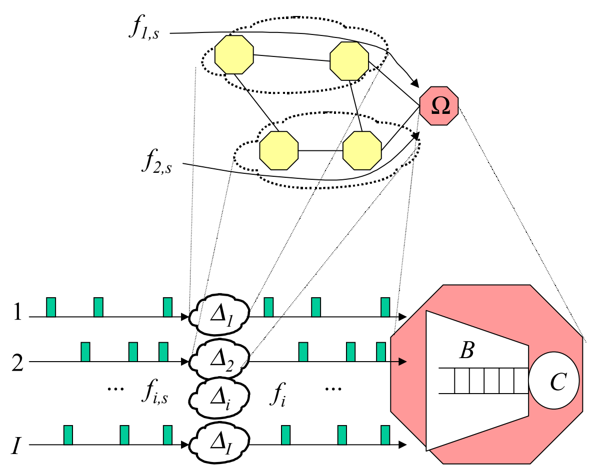

Fig. 1. Network Node Model.

In Fig. 1 *C* and *B* are the output link capacity (bit/s) time and the buffer size (bits) respectively. $\tau = L/C$ will denote the packet service time. The generic source flow $f_{i,s}$ is assumed to be an on/off process, with constant emission rate during the on state and general distribution of active period duration. Fixed packet size is assumed. For $f_{i,s}$ we assume to know:

- $T_{i,s}$: the minimum inter-departure time during the active periods;
- $\overline{T_{i,s}}$: the average inter-departure time in the long term.

The parameters $T_{i,s}$ and $\overline{T_{i,s}}$ equal the inverse of the peak and average packet rates respectively. Given that Δ_i is known (it trivially equals the sum of the queues depletion times along the path from the *i*-th source to node Ω), one can compute the minimum interarrival time for the process f_i:

$$T_i = \max\left(T_{i,s} - \Delta_i, L/C_{in}\right) \quad \textbf{(1)}$$

wherein C_{in} denotes the capacity of the input line to Ω. The average interarrival time for the process is substantially preserved along the flow path, i.e. $\overline{T_i} \cong \overline{T_{i,s}}$, as the average packet rate is affected exclusively by the loss events, whose probability should be kept small (typically not larger than 10^{-4}). One important parameter of the aggregate arrival process to Ω is the minimum interarrival time between consecutive packets of the same flow, which will denoted by $D = min_i\{T_i\}$. Another parameter of interest is the maximum number of arrivals to node Ω in a generic interval of duration W from flow f_i, denoted by $A_i(W)$: it can be easily computed from the source parameter $T_{i,s}$ (and eventually from the maximum burst size in case of sources constrained by a Token Bucket (b,p,r)) and the maximum cumulated queuing delay Δ_i. As an example, here we give the expressions of $A_i(W)$ for a generic on/off source with unlimited maximum burst size (e.g. markovian on/off):

$$A_i(W) = \max\{k : k \cdot T_i - \Delta_i < W\} = \lceil (W + \Delta_i)/T_i \rceil \quad (2)$$

and for an extremal on/off token-bucket-constrained source for the case $W<bp/(p-r)r$:

$$A_i(W) = \min\{\lceil (W + \Delta_i)/T_i \rceil, MBS / L\} \quad (3)$$

In the rest of the paper our aim will be to find an upper bound for the loss probabilities π_i $(i=1,..I)$ and π_{tot} defined as follows:

– π_i is the per-flow loss probability for the generic flow f_i, i.e. the probability that the system can not accommodate a new packet at time *t conditioned* to an arrival at time *t* from the *i*-th flow.
– π_{tot} is the loss probability at node Ω for the aggregate π_{tot} as a whole, i.e. the probability that the system can not accommodate a new packet at time *t conditioned* to an arrival at time *t* (without distinguishing the flow originating the arrival).

In general π_{tot} equals the average of the $\{\pi_i\}$ weighted over the flow average rates R_i, formally ($R = \Sigma_i R_i$):

$$\pi_{tot} = \frac{\sum_{i=1}^{I} R_i \cdot \pi_i}{R} \quad (4)$$

2.2 Case I: Direct Analysis

Under the following condition, which relates the number of different flows with the minimum interarrival time:

$$I \le D/\tau = min_i\{T_i\}/\tau \quad (5)$$

the formula derived in [1 pp. 397÷404] and [2] for the *modulated N·D/D/1 queue* can be directly applied to derive the following upper bound for the generic π_i for the system under study (with a finite buffer of size B and output capacity C):

$$\pi_i \le \sum_{N=0}^{I} P_D(N) \cdot Q_{D,\tau}^N(x) \Big|_{x=B/C} \tag{6}$$

wherein $Q^N_{D,\tau}(x)$ is the probability of having an amount of backlog larger than $x \cdot C$ in an infinite buffer at the generic instant *t given* the number of arrivals in $[t-D,t)$ is N:

$$Q_{D,\tau}^N(x) \le \sum_{\frac{x}{\tau}<n\le N} \binom{N}{n} \cdot \left(\frac{n\tau - x}{D}\right)^n \cdot \left(1 - \frac{n\tau - x}{D}\right)^{N-n} \frac{D - N\tau + x}{D - n\tau + x} \tag{7}$$

wherein $\tau = L / C$ is the packet service time and $P_D(N)$ is the probability to collect N arrivals in $[t-D,t)$. Denoting by $g_{D,i}(k)$ the probability of k arrivals from flow f_i in $[t-D,t)$, $P_D(N)$ is given by the convolution of the single $g_{D,i}(k)$ as the flows are independent, formally:

$$P_D(N) = conv\{g_{D,i}(k),\ i=1,..I\} \tag{8}$$

As D is smaller than T_i, the generic $g_{D,i}(k)$ is defined only in $k \in \{0,1\}$, precisely:

$$g_{D,i}(k) = \begin{cases} D/\overline{T_i} & k=1 \\ 1 - D/\overline{T_i} & k=0 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Unfortunately, condition (5) is quite restrictive, so that the bound (6) is not applicable in a wide range of cases of practical interest. The scope of the following section is to derive an upper bound for the cases where condition (5) is not met.

It is useful to point out that the queue analysis found in [1] and [2] which led to bound (6) is based on the following properties of the overall arrival process with reference to a generic interval θ_D of duration D:

i. at most one arrival from each flow in θ_D;
ii. the arrivals are randomly distributed in θ_D;
iii. the maximum number of arrivals in θ_D is not larger than the number of packets that can be served in the same interval.

Note that:

- property *i)* derives from the definition of D;
- property *ii)* derives from *i)* and from the independence of the flows;
- property *iii)* derives from *i)* and (5).

Finally, from (4) it derives that the right-hand term of (6) is also an upper bound for the aggregate loss probability π_{tot}.

2.3 Case II: The Analysis in Presence of a Dropper with Short Dropping Window

Condition (5) can heavily limit the number of flows that can be multiplexed. Condition (5) implicitly implies that $D > \tau$. Now we face the case where for the input traffic aggregate the condition $D > \tau$ still holds but the number of multiplexed flows is larger than D/τ, i.e. condition (5) is not met. In this case, by inserting a device called dropper$_W$ before the FIFO queue, we can refer to the system in Fig. 2, denoted by $S(W,\{f_i\})$.

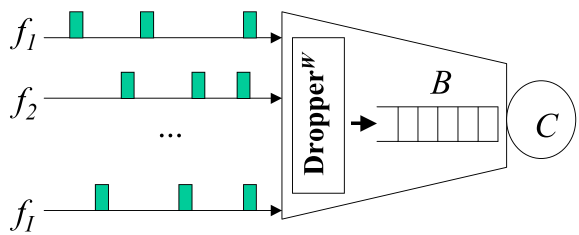

Fig. 2. The system $S(W,\{f_i\})$.

The dropper$_W$ admits a packet arriving at time t if and only if the number of packets admitted in $[t-W,t)$ is smaller than W/τ-1, otherwise it discards the packet. The duration of the dropping window is indicated as a subscript in "dropper$_W$" to stress that such a parameter characterizes the device. Under the following condition:

$$W \le D = min_i\{T_i\} \tag{10}$$

the following single upper bound can be given for the generic π_i ($i =1,..I$) in the system $S(W,\{f_i\})|_{W\le D}$:

$$\pi_i \le \sum_{N=0}^{N_d-1} P_W(N)\cdot Q_{W,\tau}^N(x)\Big|_{x=B/C} + \sum_{N=N_d}^{I} P_W(N) \tag{11}$$

wherein $N_d = \lfloor W/\tau \rfloor$ while $P_W(N)$ and $Q^N{}_{W,\tau}(x)$ are given by equations (7) through (9) by substituting W to D. This result is equivalent to that found in [3]. Demonstration of (11) is given in *Appendix A*. Substantially, the first and second terms in (11) account for the loss probability at the queue and at the dropper$_W$ respectively. It can be seen that the term relevant to the loss due to the queue is similar to (6). In facts the queue analysis for the two cases is substantially the same, as also in case II the arrival process *to the queue* holds the same properties *i)* through *iii)* defined above with reference to a generic interval θ_W of duration W. As a difference with the case I, in case II property *iii)* is a consequence of the dropper action - rather than of condition (5). It should be clear that the presence of the dropper$_W$ along with condition (10) have replaced condition (5). Finally, it can be noted that when $W = D \ge I\cdot\tau$ the bounds (11) and (6) are equivalent.

The meaning of the presence of dropper$_W$ will be discussed in section 2.5.

2.4 Case III: The Analysis in Presence of a Dropper with Long Dropping Window

The aim of this section is to provide an upper bound for the loss probability in the system of Fig. 2 (i.e. in presence of the dropper$_W$) in case condition (10) is not met, i.e. the dropping window length is longer than $D = min_i\{T_i\}$. This system will be denoted by $S(W,\{f_i\})|_{W>D}$: In this case it is not possible in general to identify any reference interval for which the above mentioned properties *i)* and *ii)* hold. In facts in any time window θ_W of duration W it is possible to collect more than one arrival from the same j-th flow, provided that $T_j < W$. That has the following consequences:

a) the distribution $g_{W,j}(k)$ of the number of arrivals from the j-th flow in the generic interval $[t\text{-}W,t)$ is unknown; in facts in general $g_{W,j}(k) >0$ also outside the set $k \in \{0,1\}$, then from $E\{g_{W,j}(k)\}$ only is not possible to univocally identify $g_{W,j}(k)$ for any k.

b) the arrivals in the generic interval $[t\text{-}W,t)$ are not uniformly distributed, as the epochs of arrivals from the same flow are not independent.

From a) it derives that the distribution $P_W(N)$ of the arrivals in $[t\text{-}W,t)$ can not be computed, while from b) it derives that the expression (7) for $Q^N_{W,\tau}(x)$ can not be applied. In order to find an upper bound for the generic π_i in $S(W,\{f_i\})|_{W>D}$, we will build a virtual scenario S_i^* such that:

- in S_i^* the per-flow loss probability for i-th flow π_i^* can be analytically upper bounded;
- S_i^* is "worse" than $S(W,\{f_i\})|_{W>D}$ in terms of the per-flow loss probability for i-th flow i.e. $\pi_i \le \pi_i^*$.

In this strategy, building up a "worse" system S_i^* is somewhat critical: roughly, our proposal is to consider S_i^* to be derived from $S(W,\{f_i\})|_{W>D}$ by substituting a "worse" flow f_j^* to each actual flow f_j ($1 \le j \le I$, $j \ne i$), and assuming a "worst case" arrival pattern for f_i in $[t\text{-}W,t)$. Denote by m_j ($1 \le j \le I$) the maximum number of arrivals that can be collected in $[t\text{-}W,t)$ from flow f_j, and by $M = max_j\{m_j\}$. When focusing on the loss probability π_i for a packet arriving at epoch t from flow f_i we will build the worse system S_i^* in the following way:

1. replace each flow f_j ($1 \le j \le I$, $j \ne i$) with a "worse" flow $f_j^*(M)$ with the same average rate in the long term but for which arrivals occur only in batches of size M packets (it is equivalent to assume packets of fixed size $M \cdot L$)
2. similarly, replace flow f_i with a "worse" flow $f_i^*(m_i)$ with the same average rate but for which arrivals occur in batches of size m_i.

The system S_i^* can be ideally realized in the way depicted in Fig. 3: before entering the multiplexer, each packet flow passes a "re-packing" stage, of parameter M for f_j ($1 \le j \le I$, $j \ne i$) and m_i for f_i. The "re-packing" stage of size k (k positive integer) has the function to gather the packets of a single flow in batches of size k: it buffers the input packets until it collects k ones, then sends them consecutively in a single batch. As $k \ge 1$, the "re-packing" stage concentrates the activity periods of the flows. Considering that the single packet *flows are randomly phased*, *in the average* the system S_i^* will present loss probabilities greater than $S(W,\{f_i\})|_{W>D}$, in other words it is statistically "worse" in terms of loss. System S_i^* is similar to the system considered in section 2.3 (case II) as all the batch arrivals in $[t\text{-}W,t)$ belong to different flows, thus are independent. Our approach will be to consider each batch of M arrivals as the arrival of a single packet with service time $\tau \cdot M$ in order to apply (7).

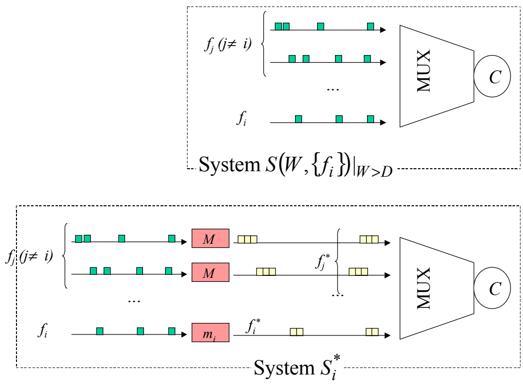

Fig. 3. System with batch arrivals.

Let's cluster the set of flows f_j according to the values of m_i, by defining the *h-class* F_h ($1 \le h \le M$) as the set of flows for which $m_i = h$, i.e. $F_h=\{f_i: m_i=h\}$. Following an approach similar to (11), for any flow of class F_h the following bound can be derived:

$$\pi_i \le \sum_{k=0}^{N_d^M-1} P_W^*(k) \cdot Q_{W,\tau\cdot M}^k(x)|_{x=\frac{(B+L-hL)}{C}} + \sum_{k=N_d^M}^{I} P_W^*(k) = \Pi_W(h) \quad \textbf{(12)}$$

wherein $N_d^M = \lfloor (N_d - h)/M \rfloor + 1$ and

$$P^*_W(k) = conv\{g^*_{W,j}(k), j=1,..I\} \quad \textbf{(13)}$$

$$g^*_{W,j}(k) = \begin{cases} W/(\overline{T_j} \cdot M) & k=1 \\ 1 - W/(\overline{T_j} \cdot M) & k=0 \\ 0 & \text{otherwise} \end{cases} \quad \textbf{(14)}$$

Demonstration is given in appendix B. Note that (12) includes (11) as a particular case when M = 1. Similarly to (11) the first and second terms in (12) account for the loss at the queue and at the dropper$_W$ respectively. The multiplicity of bounds expressed by (12) is equal to the number of different classes. Our aim is now to derive a single bound for the loss probability π_{tot} relevant to the input aggregate as a whole. Having in mind equation (4) the following bounds can be derived from (12):

1. averaging (12):

$$\pi_{tot} \leq \frac{\sum_{h \in H} R_h \cdot \Pi_W(h)}{R} \quad \textbf{(15)}$$

wherein R_h represents the sum of the mean rates for all the flows in the class F_h and H represents the set of values for h.

2. finding the maximum of (12):

$$\pi_{tot} \leq \max_{h \in H}(\Pi_W(h)) = \Pi_W(h)\big|_{h=M} \quad \textbf{(16)}$$

The bound (15) is tighter than (16), although the last one is more immediate to compute. Note that for the homogeneous scenario where only one class is present ($m_i = M \ \forall i$) the two bounds are equivalent.

2.5 Implementation Aspects

As the right-hand term in (6) is not dependent on i, it can be shown that it is an upper bound for the aggregate loss probability π_{tot} as well as for each single π_i ($i=1,..I$). The same applies to bound (11). Thus, for the cases I and II upper bounds for π_{tot} are available. Additionally, (15) and (16) provide bounds for π_{tot} for the case III. So far we have explored the cases II and III by assuming that a dropper acting on a time window of length W were present before the queue (Fig. 4b).

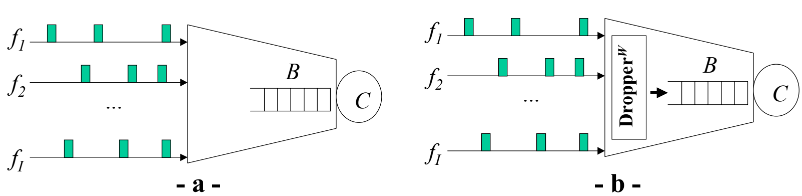

Fig. 4. Reference systems.

It must be noted that the multiplexing system of Fig. 4b is *not* work-conserving because of the presence of the dropper$_W$. As a consequence, the aggregate loss for such a multiplexer fed with a input traffic mix $\{f_i\}$, denoted by $\pi_{tot}^{(b)}$, can *not* be smaller than the aggregate loss for a multiplexer simply constituted by the FIFO queue (Fig. 4a) fed with the same input traffic mix, denoted by $\pi_{tot}^{(a)}$. Formally:

$$\pi_{tot}^{(a)} \leq \pi_{tot}^{(b)} \quad \textbf{(17)}$$

As a consequence, all the bounds for the aggregate loss found above for the cases II and III in presence of a dropper$_W$ work also for the simple FIFO queue scheme. Unfortunately, the same does *not* apply for the single *per flow* loss, i.e. in general one can *not* be sure that $\pi_i^{(a)} \leq \pi_i^{(b)} \ \forall i$.

In a real scenario, one can choose not to implement the dropper device: it only acts as a "virtual device" useful to derive the bounds for the *aggregate* loss probability. In this context the duration W of the dropper window could be arbitrarily chosen in order to minimize the bound to the loss probability. In particular, given $D = min_i\{T_i\}$ if $I \leq D$ (i.e. condition (5) holds), (6) can be used to evaluate the bound to the loss probability and there is no need to use the dropper and to optimize W. Conversely, if $I > D$ one should optimize the choice of W to obtain the tightest upper bound. The effect of choosing different values of W is depicted in Fig. 5. This figure plots the bound to the loss probability evaluated using (12). The two components of the bound relevant to the dropper and to the queue are represented. The bound is evaluated for a homogeneous scenario where a set of 120 sources is multiplexed at a generic stage (parameters are shown in Tab. 1), and the cumulative maximum delay which has been encountered by each source is $\Delta=50$ *ms*. For this case $T_{i,s} = 72$ ms, while $D = min_i\{T_i\} = T_{i,s} - \Delta = 22$ *ms*. Increasing W, the discontinuities in the curve occur when the maximum number of arrivals m that can be collected in the window of duration W increases. The minimum bound of the loss probability is achieved for $W \cong 94$ *ms*.

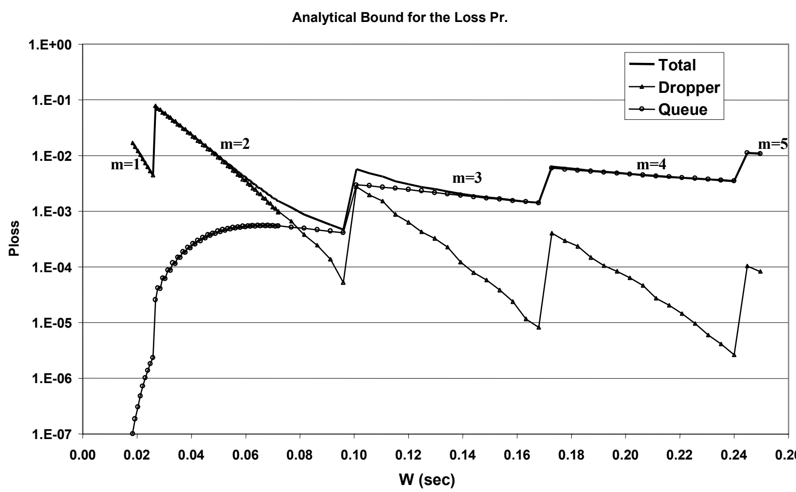

Fig. 5. Analytical bound vs. W (Δ=50 ms).

As an alternative to the approach of considering the dropper$_W$ as a virtual device, one can choose to actually implement it in the multiplexer. The *cons* are that for each dropper the dropping window duration, i.e. the parameter W, must be somehow managed, either statically or dynamically according to the mix of input flows. The *pros* of such a choice is that the bounds found previously for the *per flow* loss probabilities can be exploited *to guarantee a target maximum loss to the single flows rather than simply to the aggregate as a whole*. This would be a major advantage from a theoretical point of view, although in practice it is often sufficient to be able to control the aggregate loss.

3 Simulations and Numerical Results

Simulations have been run in order to validate the upper bounds provided above. Here for sake of simplicity we present exclusively the results relevant to the bound of case III (eq. (12)). Further simulations relevant to case II are available in [3].

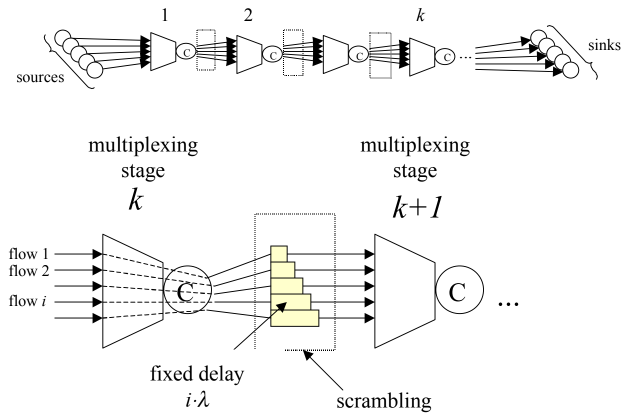

Fig. 6. Simulation scenario

The simulation scenario consists of a linear chain of identical multiplexing stages with the same output capacity and buffer size (see Tab. 1). A set of identical and independent packet sources with is attached directly to the 1st multiplexing stage. Before entering the *k+1* (*k=1,2..*) multiplexing stage, the flows are scrambled in order to eliminate the correlation in time introduced by the previous *k* stages: a fixed delay $i \cdot \lambda$ is added to the packets of the generic *i*-th flow, with λ large compared to the average burst duration. This way the flows in input to the generic multiplexing stage *k* present identical statistical properties as *i)* were originated by identical sources and *ii)* experienced identical jitter processes as passed through the same previous multiplexing processes. This simulation technique was also used in [7].

Table 1. Simulation parameters

Link capacity $C = 5\ Mb/s$	Peak rate $P = 64\ Kb/s$
Buffer size $B = 10 \cdot L$	Activity $a = 0.4$
Packet size $L = 576 bytes$	$T_{on} = 1.2\ sec$, $T_{off} = 1.8\ sec$

Two kinds of on/off sources were considered (parameters are shown in Tab. 1):

- **Markovian sources**: with *average* duration of active and idle period T_{on} and T_{off} respectively;
- **Extremal TB sources**: with active and idle periods of *fixed* duration T_{on} and T_{off} respectively: such sources are representative of extremal on/off sources con-

strained by a token bucket with parameters p = 64 Kb/s, r = 25.6 Kb/s, b = 5760 bytes.

All the simulations confirmed that the empirical loss stays below the analytical bound at each multiplexing stage. For sake of simplicity only the curves relevant to 3rd multiplexing stage in the case of the Extremal TB sources are presented. Two sets of simulations have been run: in the first set the dropper$_W$ device was actually implemented at each stage, i.e. the multiplexer scheme was that of Fig. 4b. The dropping window duration was set at $W = 84$ ms, which resulted in a value of $m_i = 2\ \forall i$ at the 3rd stage, as can be derived from eq (3) considering that the maximum queuing delay introduced by the two previous multiplexing stages is $\Delta = \Delta_i = 18.4$ ms $\forall i$ and the minimum inter-departure time is $T = T_i = 72$ ms $\forall i$. Fig. 7 shows the empirical percentage of packet lost in the whole multiplexer with the analytical bound given by (12) (only one class is present) for different loads. The load was varied by varying the number I of multiplexed flows. Note that the abscissa axis reports the *average* load, i.e. $\rho_{av} = I \cdot P \cdot a/C$. Fig. 7 also compares the empirical percentage of packets lost at the *queue* and at the *dropper* respectively with the first and second terms of (12).

Fig. 7 clearly shows that the bound is met in the whole range of considered loads. Moreover it is evident that the analytical curves follow quite well the behavior of the empirical ones.

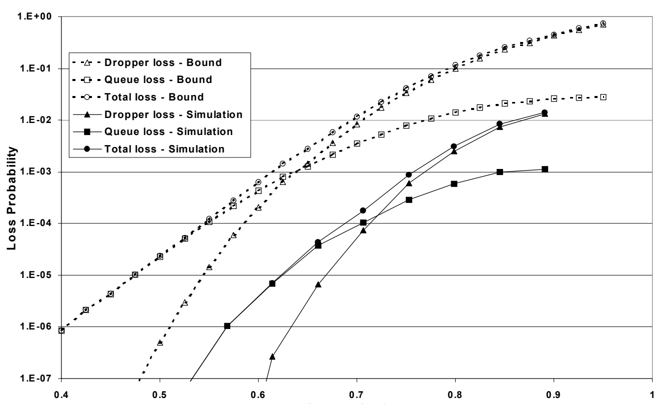

Fig. 7. Comparison between bounds and simulation (1)

In the second set of simulations the dropper$_W$ was not implemented at all, i.e. the multiplexer scheme was the simple FIFO queue of Fig. 4a. Fig. 8 shows the empirical percentage of packet lost vs. the average load, and compares them with analytical bounds computed for different values of parameter W. It can be seen that the empirical curve remains below the analytical bounds for all the considered values of parameter W.

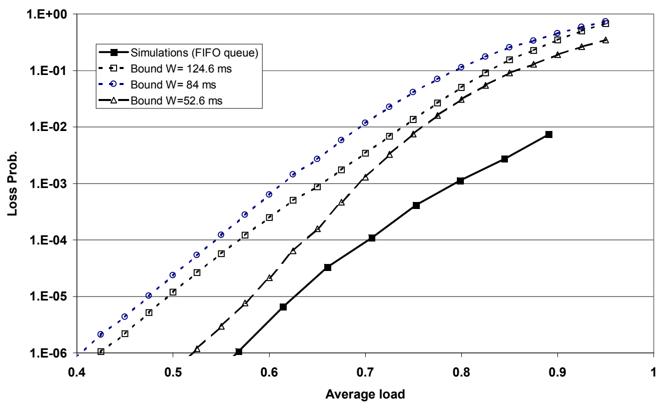

Fig. 8. Comparison between bounds and simulation (2)

4 Application to a Case Study

Throughout this paper we derived upper bounds for the loss probability at a generic network node. The effect of jitter introduced by previous multiplexing stages, supposed non negligible, has been properly taken into account. Such bounds can be used to perform Admission Control, in order to provide a service with loss guarantees in a Diffserv network. In order to gain an insight about the efficiency of an Admission Control scheme based on our bounds, we computed the achievable utilization efficiency for the following sample case. Consider a cascade of multiplexers with output capacity $C = 10$ *Mb/s* and buffer size $B = 30$ packets, fed by on/off packet flows with packet size $L = 576$ bytes, peak rate $P = 128$ *Kb/s* and activity $a = 0.7$. Denote by Ω_{K+1} the generic ($K+1$)-th multiplexer along the cascade: the maximum variable delay (jitter) introduced by the previous multiplexing stages onto the flows entering Ω_{K+1} is $\Delta_{K+1} = K \cdot B \cdot L/C$. Given a maximum admitted loss probability $\Pi = 10^{-4}$, we computed the maximum number of flows that can be admitted to cross Ω_{K+1} so that the bound (16) stays below the threshold Π. Tab. 2 shows the results for different values of K, along with the resulting *average* utilization efficiency $\rho_{av} = I_{max} \cdot P \cdot a/C$.

Table 2. Maximum number of flows and achievable average load for a sample case ($\Pi = 10^{-4}$).

K	1	2	3	4	5	6	7
I_{max}	82	74	72	66	59	58	52
ρ_{av}	0.73	0.66	0.65	0.59	0.53	0.52	0.47

It can be seen from Tab.2 that the achievable efficiency decreases with the multiplexing stage due to the effect of jitter, but the decrease is not dramatic as for the worst-case based admission control schemes [4], which present much lower efficiency. The values of achievable efficiency provided in Tab. 2 suggest that the proposed bounds are feasible to be used in Admission Control schemes of practical interest.

5 Conclusions and Future Works

We have presented a statistical approach to bound the loss probability in a network multiplexer fed by jittered flows. A set of simulations have been run in order to validate the bounds. Some numerical results have been presented, which show that such bounds can be used to perform Admission Control in a more effective way than by relying on deterministic worst-case approaches.

As directions for further study we envisage extensions to the proposed approach in order to *i*) deal with variable size packets and *ii*) take into account the effect of priority scheduling.

Appendix A

We want to derive an upper bound for π_i, i.e. the probability that the system is not able to accommodate (i.e. discards) an arrival at time t *conditioned* to the event Φ_i = "one packet arriving from flow f_i at epoch t". Consider that a packet arriving at t can be discarded either at the dropper$_W$ if "the number of packets already admitted in $[t\text{-}W,t)$ equals $N_d = \lfloor W/\tau \rfloor$" (denote such an event by Θ_d) *or* at the queue if "the backlog in t^- is larger than B" (denote such an event by Θ_q)[1]. Using this formalism we can write $\pi_i = Pr\{\Theta_d \cup \Theta_q \mid \Phi_i\}$. By partitioning on the number of arrivals in $[t\text{-}W,t)$, denoted by A_W, we can write:

$$\pi_i = \Pr\{\Theta_d \cup \Theta_q \mid \Phi_i\} = \sum_N \Pr\{\Theta_d \cup \Theta_q \mid \Phi_i, A_W = N\} \cdot \Pr\{A_W = N \mid \Phi_i\} \le \quad (18)$$
$$\le \sum_N \Pr\{\Theta_d \cup \Theta_q \mid \Phi_i, A_W = N\} \cdot \Pr_W(N)$$

with $P_W(N) = Pr\{A_W = N\}$. The last inequality holds as the arrival at epoch t from flow f_i excludes the possibility to collect one arrival from c itself in $[t\text{-}W,t)$ as $W \le T_i$, while the arrivals from the other flows f_j ($j \neq i$) are independent from Φ_i: then it derives that substituting $Pr\{A_W = N\}$ in place of $Pr\{A_W = N \mid \Phi_i\}$ leads to an upper bound. Further split the sum in (18) into two, for $0 \le N \le N_d\text{-}1$ and $N_d \le N \le I$, and consider that the event Θ_d has null probability when $A_W < N_d$. We end up with the following inequality:

$$\pi_i \le \sum_{N=O}^{N_d-1} \Pr\{\Theta_q \mid \Phi_i, A_W = N\} P_W(N) + \sum_{N=N_d}^{I} \Pr\{\Theta_d \cup \Theta_q \mid \Phi_i, A_W = N\} P_W(N) \quad (19)$$

As the arrival at time t does not influence the queue state at t^- the term $\Pr\{\Theta_q \mid \Phi_i, A_W = N\}$ in the first sum can be rewritten as $\Pr\{\Theta_q \mid A_W = N\}$: it represents the probability to have an amount of backlog larger than B at t *conditioned* to N ($N < W\cdot\tau$) arrivals in $[t\text{-}W,t)$ for a finite buffer system, and can be bounded by the same probability for the case of infinite buffer, i.e. $Q^N_{W,\tau}(x)|_{x=B/C}$ as given by eq. (7) with W in place of D. Note that eq. (7) can be applied as the arrivals in $[t\text{-}W,t)$ are

[11] Note that a service system with fixed packet size L and a buffer of size B can contain up to $B+L$ work: B in the queue plus L in the servent.

independent (they all belong to different flows) and then uniformly distributed in [*t*-*W,t*). Finally, the term $\Pr\{\Theta_d \cup \Theta_q \mid \Phi_i, A_W = N\}$ in the second sum can be roughly bounded by 1. From (19) we can thus derive (11).

Appendix B

Let's consider a flow f_i belonging to the class F_h, which means that the maximum number of arrivals that can be collected in [*t*-*W,t*) from flow f_j is $m_i = h$. As we have no information about the pattern of such arrivals, i.e. the correlation between the arrival epochs, we consider the "worst case" scenario in which such correlation is maximal, i.e. the h arrivals occur at the same epoch t. Denote by Φ_i^h the event "one batch of size h arrives from flow f_i at epoch t". The loss probability π_i can be expressed as follows:

$$\pi_i = \sum_{r=0}^{h} \frac{r}{h} \cdot P_r \le \sum_{r=1}^{h} P_r \qquad \textbf{(20)}$$

wherein P_r represent the probability that r packets are discarded over the h ones constituting the batch arrival from f_i. Equality holds for $h = 1$. In a fashion similar to Appendix A, denote by Θ_d^h the event "the number of packets already admitted in [*t*-*W,t*) equals $N_d - h$" (where $N_d = \lfloor W / \tau \rfloor$) and by Θ_q^h the event "the backlog in t^- is larger than $B + L - h{\cdot}L$", i.e. the buffer can not accommodate all the incoming h packets. Clearly the sum in the last term of (20) represents the probability that *at least one* packet is discarded - either at the dropper or at the queue - *conditioned* to event Φ_i^h, then using the formalism introduced above we can write from (20):

$$\pi_i \le \Pr\{\Theta_d^h \cup \Theta_q^h \mid \Phi_i^h\} \qquad \textbf{(21)}$$

Following the same procedure used for (18) in Appendix A, (20) can be partitioned on the number A^*_W of arrivals in [*t*-*W,t*). Note that in the system under study the number of arrivals in [*t*-*W,t*) is an integer multiple of M, i.e. $A^*_W = k{\cdot}M$ with k integer. Then we obtain:

$$\begin{aligned}\pi_i &\le \sum_{k=O}^{I} \Pr\{\Theta_d^h \cup \Theta_q^h \mid \Phi_i^h, A_W^* = k \cdot M\} \cdot \Pr\{A_W^* = k \cdot M \mid \Phi_i^h\} \le \\ &\le \sum_{k=O}^{I} \Pr\{\Theta_d^h \cup \Theta_q^h \mid \Phi_i^h, A_W^* = k \cdot M\} \cdot \Pr\{A_W^* = k \cdot M\}\end{aligned} \qquad \textbf{(22)}$$

Further split the sum into the ranges where $0 \le A^*_W \le N_d\text{-}h$ (or equivalently $0 \le k \le (N_d\text{-}h)/M$) and $N_d - h + 1 \le A^*_W \le I{\cdot}M$ (or equivalently $(N_d\text{-}h)/M < k \le I$). Considering that the event Θ_d^h has null probability when $A_W \le N_d - h$, we end up with the following equation (similar to (19)):

$$\pi_i \le \sum_{k=O}^{N_d^M - 1} \Pr\{\Theta_q^h \mid \Phi_i, A_W^* = k \cdot M\} P_W^*(k) + \sum_{k=N_d^M}^{I} \Pr\{\Theta_d^h \cup \Theta_q^h \mid \Phi_i, A_W^* = k \cdot M\} P_W^*(k) \qquad \textbf{(23)}$$

wherein $N_d^M = \lfloor (N_d - h)/M \rfloor + 1$ and $P^*_W(k)$ has replaced $\Pr\{A^*_W = k \cdot M\}$. Like for (19) the term $\Pr\{\Theta_q^h \mid \Phi_i, A_W^* = k \cdot M\} = \Pr\{\Theta_q^h \mid A_W^* = k \cdot M\}$ in the first sum represents the probability to have an amount of backlog larger than $B + L - h \cdot L$ at t^- *conditioned* to k arrivals (of batches with M packets) in $[t-W,t)$ for a finite buffer system. To find an upper bound for it, we can apply eq. (7) by considering that a batch of M arrivals is equivalent to the arrival of a single packet with service time $\tau \cdot M$ and that different batch arrivals belongs to different flows and thus are independent. Formally we have for $0 \le k \le N_d^M - 1$:

$$\Pr\{\Theta_q^h \mid \Phi_i, A_W^* = k \cdot M\} \le Q_{W,\tau \cdot M}^k(x)\Big|_{x=\frac{(B+L-hL)}{C}} \quad \textbf{(24)}$$

Finally, the term $\Pr\{\Theta_d^h \cup \Theta_q^h \mid \Phi_i, A_W^* = k \cdot M\}$ in the second sum can be roughly bounded by 1. Thus from (24) we derive (12).

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