REVISED UPSTREAM POWER BACK-OFF FOR VDSL

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ABSTRACT

Accurate upstream power back-off (PBO) parameters are needed by operators deploying very high-speed digital subscriber line (VDSL) modems. Although a standardized PBO method for VDSL exist, the standard gives little or no guidance to an operator how to establish these optimized PBO parameters for its particular network and customers. In this paper, we present an efficient algorithm based on the Nelder-Mead simplex search which calculates optimized upstream PBO parameters. To make the PBO parameter calculation independent of the network scenario we present a new method for establishing worst-case far-end crosstalk (FEXT) noise, which is based on virtual modems.

1. INTRODUCTION

Very high-speed digital subscriber line (VDSL) is one of the latest introduced DSL technology, which currently utilizes frequencies up to 12 MHz. It uses frequency division duplex (FDD) transmission scheme in order to avoid near-end crosstalk (NEXT) noise between VDSL systems. Furthermore, for robustness reasons, current standardized VDSL systems use two frequency bands for each transmission direction, i.e., four band plans are employed.

Many PBO methods have been proposed for VDSL, as described by Schelstraete in [1] and the references therein. However, standardization bodies have agreed to use the reference PBO method [2] where different reference PSDs have been defined for each upstream band. The actual parameters used for the reference PBO in the current VDSL standards were established by Schelstraete [1] and Oksman [3]. They both used a kind of exhaustive search to find optimized PBO parameters, which is time consuming. To circumvent this problem, we show how to calculate the PBO parameters by using the Nelder-Mead simplex algorithm [4].

To make the calculation of PBO parameters independent of the network scenario a worst-case far-end crosstalk (FEXT) noise concept has been introduced [1]. However, we have discovered that this concept does not always represent the worst-case, especially for discrete multi-tone (DMT) based VDSL systems. Therefore, we present a new improved way to establish the worst-case FEXT noise, which is based on a concept of virtual modems.

The paper is organized as follows: Section 2 briefly describes some preliminaries concerning VDSL systems; Section 3 shows our improved method to calculate the worst-case FEXT noise; Section 4 presents the proposed algorithm to find the optimized PBO parameters; and Section 5 summarizes the major findings in this paper.

2. PRELIMINARIES

The upstream bitrate of a VDSL system is calculated, based on Shannon’s formula, as

\[
R = \int_{f_U} \log \left(1 + \frac{\text{SNR}(f)}{\Gamma}\right) df,
\]

where \( f \) is the frequency, \( f_U \) is the set of frequencies used in the upstream direction, \( \Gamma \) is the signal-to-noise ratio (SNR) gap, and \( \text{SNR}(f) \) is the received signal-to-noise ratio. The SNR can be expressed as

\[
\text{SNR}(f) = \frac{P_{\text{Rx}}(f)}{P_{\text{TotN}}(f)} = \frac{|H(f)|^2 P_{\text{Tx}}(f)}{P_{\text{F}}(f) + P_{\text{BGN}}(f)},
\]

\[\text{(1)}\]

\[\text{(2)}\]
where $P_{\text{Rx}}(f)$ is the received signal PSD, $P_{\text{TotN}}(f)$ is the total noise PSD at the receiver. $P_{\text{Tx}}(f)$ represents the transmit PSD, $|H(f)|^2$ is the cable insertion loss, $P_{\text{F}}(f)$ is the FEXT noise from other VDSL systems, $P_{\text{BGN}}$ consists of alien noise and any other type of background noise. The NEXT noise is avoided due to the FDD transmission. The FEXT noise depends on the modems transmit PSDs and the FEXT crosstalk couplings, which are typically quite random in nature.

However, within VDSL standardization a conservative 99% worst-case crosstalk coupling model is used

$$P_{\text{F}}(f) = K_F N^{0.6} f^2 l_x |H(f,l)|^2 P_{\text{Tx}}(f),$$

where $l_x$ is the coupling length, $N$ is the number of disturbing VDSL modems, and $|H(f,l)|^2$ is the insertion loss of disturbing modems. The constant $K_F$ is empirically determined by ETSI to be $-45 \text{ dB}$ at $1 \text{ MHz}$, when the frequency $f$ is expressed in MHz and the length $l$ in km [2].

### 2.1. The Reference PSD Method for PBO

The reference PSD method was developed after observing that many PBO methods could be described by a certain desired received PSD. This reference PSD, $P_R(f)$, which determines the maximum received PSD, is a parameterized function of frequency that can be designed to meet certain objectives. One such common objective is the maximum reach for a predefined set of bitrates. Even if almost any shape of $P_R(f)$ is conceivable, for practical reasons it was decided in the standardization process [1, 3] to select a reference PSD model expressed as

$$P_{\text{R,dbm}}(f) = \alpha + \beta \sqrt{f}, \quad [\text{dBm/Hz}], \quad (3)$$

where $f$ is given in MHz, and $\alpha$ and $\beta$ are the parameters that are free to be determined in order to maximize the reach. It was also decided that independent reference PSDs should be assigned for each upstream band.

In addition, modems need also adhere to a maximum allowed transmit PSD, $P_{\text{max}}(f)$. Hence, the transmit signal PSD of a particular user $n$ is given by

$$P_{\text{Tx},n}(f) = \min \left\{ P_{\text{R}}(f) |H(f,l_n)|^{-2} , P_{\text{max}}(f) \right\}, \quad (4)$$

The optimized reference PSDs depends on the alien noise, the maximum transmit PSD mask, cable types, the network topology, and the services (bitrates) the operator wants to offer. Their influence on the reference PSDs is analyzed in [1].

To make the reference PSDs independent of any particular network scenario Schelstraete [1] proposed to use the worst-case FEXT noise model. That is, the reach for a particular set of PBO parameters will be based on the scenario that gives the worst-case FEXT. We can then use the following cost function, which minimizes the maximum between the reach without and with PBO of all protected bitrates, to find the optimized reference PSDs:

$$y = \min \left\{ I_{\text{NoPBO}}(R_i) - I_{\text{PBO}}(R_i) \right\}, \quad (5)$$

where $R_i$ denote the bitrates for which the reference PSDs are optimized; $I_{\text{NoPBO}}(R_i)$ denotes the reach without PBO and collocated disturbers; and $I_{\text{PBO}}(R_i)$ denotes the reach with PBO and worst-case FEXT. A similar approach was used in [1, 3] to find the optimized reference PSDs.

There are of course another ways to define the optimization criteria for the cost function $y$ in (5). For instance, we can minimize the differences between $I_{\text{NoPBO}}(R_i)$ and $I_{\text{PBO}}(R_i)$ for different $R_i$ such that all bitrates are protected equally or differently based on some constraints. However, in this paper we restrict ourself to (5), since we think it is a good optimization strategy.

### 3. WORST-CASE FEXT NOISE

From (1) and (2) one can easily see that for the case when all frequencies can be utilized for transmission, the worst-case performance appears when the integral of the FEXT noise is the greatest. This worst-case FEXT noise, $P_{\text{F,WC}}$, was in [1] calculated by assuming that all disturber modems are collocated. Depending on the length of the band $l_V$ and the disturbing (collocated) modems $l_0$ the following expression for the worst-case FEXT noise has been used:

$$P_{\text{F,WC}}(f) = \begin{cases} K_F N^{0.6} f^2 l_V P_R(f) & \text{if } l_V \leq l_0 \\ K_F N^{0.6} f^2 l_0 P_R(f) & \text{if } l_V > l_0 \end{cases}, \quad (6)$$

where $l_0$ is determined by the maximum of

$$\Phi(l_i) = \int_{f_U} \phi(l_i, f) df \quad (7)$$

$$= \int_{f_U} K_F N^{0.6} f^2 l_i \min \{ P_R(f), |H(f,l_i)|^2 P_{\text{max}}(f) \} df. \quad (8)$$

Thus, $\Phi(l_i) < \Phi(l_0)$ for all $l_i \neq l_0$.

The value $l_0$ can be found by integrating (8) with $P_R = P_{\text{R,1U}}$ and $P_R = P_{\text{R,2U}}$ for the first and second upstream bands, since the reference PSDs are independently defined for both upstream bands. Superscripts 1U and 2U denote the first and second upstream bands, respectively. For instance, for $P_{\text{R,1U,dbm}} = -60 - 17 \sqrt{T}$ and $P_{\text{R,2U,dbm}} = -60 - 12 \sqrt{T}$ we have found that the length $l_0 = 612 \text{ m}$ causes the worst-case FEXT, for which length the PSD of FEXT noise is shown in Fig. 2 with dashed line. Assume now that we are searching the loop reach for a low bitrate. Due to high loop attenuation, modems will utilize, for example, only the frequencies of the first upstream band. Thus, the FEXT noise that is determining the reach depends only on the PSD of FEXT noise on the
first upstream band. When calculating the length that causes the worst-case FEXT on only the first upstream band we have found that \( l_0 = 893 \) m. For this length the PSD of FEXT noise is shown in Fig. 2 with solid line and is approximately 2 dBm/Hz above the case when \( l_0 = 612 \) m. For illustration purposes in Fig. 2 is shown also the length \( l_0 = 611 \) m that causes the worst-case FEXT only on the second upstream band.

![Fig. 2. FEXT noise from 20 disturbers for reference PSDs: \( P_{R-1U, \text{dBm}} = -60 - 17\sqrt{f} \) and \( P_{R-2U, \text{dBm}} = -60 - 12\sqrt{f} \).](image)

**Fig. 2.** FEXT noise from 20 disturbers for reference PSDs: \( P_{R-1U, \text{dBm}} = -60 - 17\sqrt{f} \) and \( P_{R-2U, \text{dBm}} = -60 - 12\sqrt{f} \).

To deal with the problems described in previous paragraph we propose to use ‘virtual modems’ which only transmit in a single upstream band and each of them is placed as a worst-case disturber in a particular band. In Fig. 3, Tx-1U and Tx-2U denote disturbing ‘virtual modems’, which transmit only in the first and second upstream bands, respectively. Now, for the two-band case the worst-case FEXT noise is calculated as

\[
\begin{align*}
P_{F-WC}(f) = \begin{cases} 
af^2lv (P_{R-1U}(f) + P_{R-2U}(f)) & \text{if } lv \leq l_{01}, l_{02} \\
af^2(l_01P_{R-1U}(f) + lvP_{R-2U}(f)) & \text{if } l_{01} < lv < l_{02} \\
af^2(lvP_{R-1U}(f) + l_{02}P_{R-2U}(f)) & \text{if } l_{02} < lv < l_{01} \\
af^2(l_01P_{R-1U}(f) + l_{02}P_{R-2U}(f)) & \text{if } lv \geq l_{01}, l_{02}
\end{cases}
\end{align*}
\]

where \( a = K_FV^{0.6} \), \( l_{01} \) and \( l_{02} \) are the lengths for which (8) achieves its maximum in the first and second upstream bands, respectively. It is worth mentioning that even this proposed scheme does not represent the true worst-case environment, since due to constraint (4), \( \Phi(l_i) \leq \Phi(l_0) \) in (7) always holds but not \( \phi(l_i, f) \leq \phi(l_0, f) \) for any \( f \). Theoretically, the network scenario that causes the worst-case noise can be built as in Fig. 3 with the number of virtual modems equal to the number of subcarriers used in the upstream and each of them transmitting only in one subcarrier. However, this will make the computation very challenging and furthermore with our proposed scheme we are very close to the network scenario that causes the worst-case FEXT noise.

### 4. THE OPTIMIZATION ALGORITHM

Previous attempts [1, 3] to find the \( \alpha \)’s and \( \beta \)’s in (3) for both upstream bands, which minimize the cost function in (5) use some form of exhaustive search. This strategy makes the search for the optimized reference PSDs very challenging and time consuming due to a large search space for \( \alpha \)’s and \( \beta \)’s. Instead, we propose to use the Nelder-Mead simplex algorithm [4] to search for \( \alpha \)’s and \( \beta \)’s. The pseudo-code of the proposed algorithm is listed as Algorithm 1.

**Algorithm 1** PBO optimization based on Nelder-Mead for two upstream bands

<table>
<thead>
<tr>
<th><strong>Initial Values</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_i ) {Bitrates to protect}</td>
</tr>
<tr>
<td>( x = [\alpha_{1U}, \beta_{1U}, \alpha_{2U}, \beta_{2U}] )</td>
</tr>
</tbody>
</table>

**Main Function**

repeat

\[ y, x = \text{NelderMead}(\text{@ReachDiff}, x) \]

until the specified accuracy have been reached

**Function** \( y = \text{ReachDiff}(R_i) \)

Find \( l_{\text{NaPBO}}(R_i) \) for all \( R_i \)

Find \( l_{01} \) and \( l_{02} \)

for each \( R_i \), do

Test \( R_{i0} \) \{Test if \( R_i \) can be achieved for zero loop length\}

if \( R_{i0} \) true then

Find \( l_{\text{PBO}}(R_i, \alpha_{1U}, \beta_{1U}, \alpha_{2U}, \beta_{2U}) \)

\{For each tested length, \( P_{F-WC} \) is calculated as in (9)\}

else

\( l_{\text{PBO}}(R_i, \alpha_{1U}, \beta_{1U}, \alpha_{2U}, \beta_{2U}) = 0 \)

end if

end for

\[ y = \min \left( \max \left\{ l_{\text{NaPBO}}(R_i) - l_{\text{PBO}}(R_i) \right\} \right) \]

The Nelder-Mead algorithm starts with the single initialization point \( X_0 = x \) which has \( D \) dimensions (for two upstream bands \( D = 4 \)) and then the Nelder-Mead simplex algorithm constructs an initial simplex with \( D + 1 \) points. The additional \( D \) points are calculated by

\[ X_d = X_0 + \lambda e_d, \quad \text{for } d = 1 \ldots D \]
where the \( e_d \)'s are \( D \) unit vectors and \( \lambda \) is a constant. For our case the search works well with any \( \lambda \) value between 0.05 and 0.1. Then, depending on the outcomes \( y \) of \textbf{ReachDiff} function, the simplex figure is changed according to the Nelder-Mead algorithm as explained in [4] until the diameter of the simplex and the difference between the two minimum values of \( y \) have reached the specified accuracies.

Since the power-sum FEXT is a concave function of loop length, the lengths \( l_{01} \) and \( l_{02} \) can be found by using the golden section search algorithm. To find the reach \( l_{\text{NoPBO}} \) the bisection line search algorithm or any another line search algorithm can be used, because for that case the SNR and therefore also the bitrate are decreasing functions of loop length. However, the simple bisection search can fail to find the reach \( l_{\text{PBO}} \), since for specific \( \alpha \)'s and \( \beta \)'s there are cases where the bitrate is a constant function of loop length. This arises for the reaches that lie above the lengths where virtual modems are placed and below the length when the received PSD begins to be lower than the reference PSD due to \( P_{\text{max}} \) constraint. For all these reaches the noise is not increased and the received signal is the same, which results in equal bitrates. If the bitrate that we search for lies exactly in that flat area the bisection will fail to find the maximum reach. However, the bisection algorithm can be extended in a straightforward way to deal with such cases.

It should also be noted that our algorithm can still find optimized PBO parameters when the worst-case FEXT is calculated without virtual modems as in (6). This is due to the fact that also for this case the bitrate is non-increasing function of loop length.

### 4.1. Simulation Results and Discussions

The proposed algorithm can be used with any number of upstream bands. However, for easy comparison with the previously published results, we have used simulation parameters according to ETSI's VDSL standard. Thus, we use \( \Gamma = 12.3 \text{ dB} \) as the SNR gap, cable TP100, and the FEXT noise from 20 disturbers. We also selected for our simulations the band plan 997, which uses two upstream bands.

Nelder-Mead algorithm is an ad-hoc optimization method, which finds the global maximum of a function if it is concave and a local maximum near the initialization point if it is non-concave. Therefore, by selecting a ‘good’ initialization point \( X_0 \) we reduce the number of iterations and find a good local maximum for non-concave functions. We have noticed that the proposed algorithm performs well if \( \alpha \)'s are initialized to the maximum PSD values and \( \beta_U \) and \( \beta_{2U} \) are initialized to the insertion losses of reaches without PBO and collocated modems for the lowest and highest bitrates, respectively.

For the case when \( \alpha \)'s are fixed and we search only for optimized \( \beta \)'s the cost function in (5) is piecewise concave (there are flat areas). This can be shown by plotting (5) for all combination of \( \beta \)'s. Furthermore, as can be seen from the bitrate/reach plots in Fig. 4 the performance for fixed and varying \( \alpha \)'s are nearly similar. Therefore, we propose to fix \( \alpha \)'s and to search only for the optimized \( \beta \)'s, since this strategy substantially reduces the number of iterations.

![Fig. 4. Upstream bitrates for noise model E for equal-length disturbers and for reference PSDs optimized to protect bitrates: 3, 6, and 12 Mbit/s.](image)

### 5. CONCLUSIONS

In this paper we presented an efficient algorithm to calculate the optimized parameters for PBO in VDSL, which uses the Nelder-Mead simplex search [4]. We also developed a new method to calculate the worst-case FEXT noise, which is especially important for DMT-based VDSL systems. The high efficiency of our algorithm allows deployment in DSL transmission systems with more than two upstream bands, which will be the case for VDSL2. This efficiency also allows operators to optimize the PBO parameters for their networks, i.e., cables, noises, and selected VDSL type.

### 6. REFERENCES


